

# Analysis and Design of Cognitive Networks: A Geometric View

Martin Haenggi



International Conference on Computer Communication Networks  
Zürich, Switzerland, August 2, 2010

Work supported by the U.S. National Science Foundation and  
the Defense Advanced Research Project Agency (DARPA)

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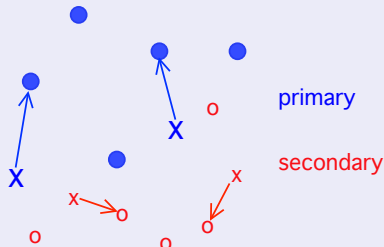
## Overview

- Background and Regulations
- Interference and the Role of the Network Geometry
- Introduction to Stochastic Geometry
- Application to TV White Space
- Application to Peer-to-Peer Networking
- Outlook and Concluding Remarks

# Cognitive Networking

## Ingredients

- A wireless network operated by an *incumbent user*
- A **secondary** or **cognitive** user who wishes to operate a network in the same frequency band
- Software-defined radios
- Maxwell's equations
- **Government regulations and spectrum policies**



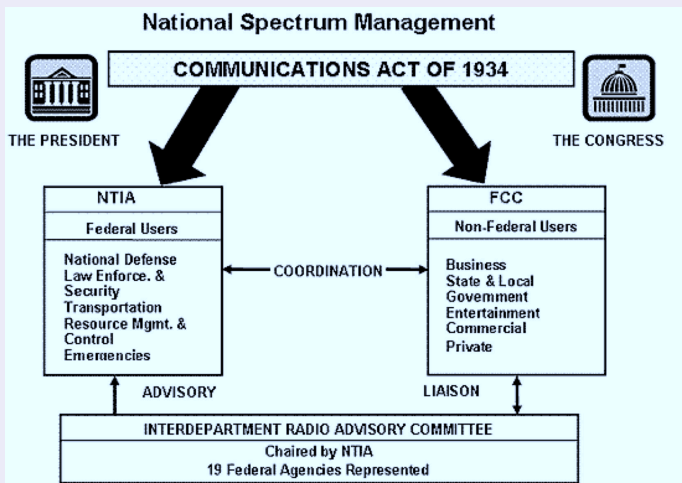
# Regulations

## US Government Agencies

- **NTIA:** National Telecommunications and Information Administration ([www.ntia.doc.gov](http://www.ntia.doc.gov)). Part of US Dept. of Commerce. Manages federal use of spectrum.  
**OSM: Office of Spectrum Management** ([www.ntia.doc.gov/osmhome/0smhome.html](http://www.ntia.doc.gov/osmhome/0smhome.html)).
- **FCC:** Federal Communications Commission ([www.fcc.gov](http://www.fcc.gov)). Manages all other uses of spectrum.  
**Wireless Telecommunications Bureau** ([wireless.fcc.gov](http://wireless.fcc.gov)).  
**Spectrum Policy Task Force** (<http://www.fcc.gov/sptf/>).

# US Spectrum Management

## Overview





## Spectrum Policy Task Force Report (Nov. 2002)

The FCC Spectrum Policy Task Force concluded in their 2002 report that:

- There is plenty of **white space**, *i.e.*, unused time or frequency slots in the TV band (channels 2–51; 54–698 MHz).
- Interference management has become more difficult due to greater density, mobility, and variability of RF transmitters; it becomes **even more problematic** if users are granted increased flexibility in their spectrum use.

## FCC National Broadband Plan ([www.broadband.gov](http://www.broadband.gov), March 2010)

### Chapter 5.6: Expanding Opportunities for Innovative Spectrum Access Models

- *Recently, the FCC has taken steps to allow **innovative spectrum access models** in the white spaces of the digital television spectrum bands and in the 3.65 GHz band. In 2006, the FCC concluded a rulemaking allowing commercial users to employ **opportunistic sharing techniques** to share 355 MHz of radio spectrum with incumbent federal government radar system operators.*
- *Using **Dynamic Frequency Selection detect- and avoid algorithms**, commercial interests are now able to operate Wireless Access Systems in the radio spectrum occupied by preexisting radar systems. **Opportunistic sharing arrangements** offer great potential to meet an increasing market demand for wireless services by promoting more efficient use of radio spectrum.*



## NTIA's Federal Strategic Spectrum Plan 2008

- *For many bands and services, NTIA envisions increased spectrum sharing through **cognitive, self-adjusting spectrum use**.*
- *Many agencies are supporting or plan to implement **SDR technologies**, which describe a new type of radio communications equipment that can automatically be reprogrammed to transmit and receive within a wide range of frequencies, using any stored transmission format. SDRs rely on embedded and programmable software for modifying and upgrading functionality and configuration. In addition, SDRs are capable of altering software based algorithms used for baseband signal processing of multiple waveform types, as well as intermediate frequency processing alternatives.*

## NTIA's Federal Strategic Spectrum Plan 2008

- *Cognitive radios are designed to be able to **perceive** and **know** the radio environment in which they are situated. The cognitive radio **senses** its environment, has the ability to track changes and react to those electro- magnetic environmental findings and **adapt** its operation accordingly. Cognitive radios can **dynamically use** whatever spectrum is available in a particular instant of time.*

## NTIA's Federal Strategic Spectrum Plan 2008 (Section B-3)

- DOD is developing programmable radio products, specifically under the Joint Tactical Radio System (JTRS) program umbrella. The JTRS is a family of modular, multi-band, multi-mode radios that will provide the basis for advanced IP-based networked communication systems.
- DOI is interested in deploying software-defined radio in the future, as an efficient way to adapt, update, and enhance a system via software upgrades.
- DOJ will pursue "smart" technologies to adaptively exploit available resources. It envisions a technical state where radio frequency systems are no longer band dependent, allowing the DOJ to **expand operations**.

## Unlicensed Access

### 2008 FCC Report and Order and Memorandum (FCC 08-260)

Permits "unlicensed operation in the TV broadcast bands" and promises "additional spectrum for unlicensed devices below 900 MHz and in the 3 GHz band". (Nov. 4, 2008).

### Accessing a database of all fixed devices

*All devices, except personal/portable devices operating in client mode, must include a geolocation capability and provisions to access over the Internet a database of protected radio services and the locations and channels that may be used by the unlicensed devices at each location.*

### Sensing

Alternatively, unlicensed users may sense the presence of primary users and transmit if they do not detect any primary transmission they could interfere with.

## Spectrum Sensing (FCC 08-260)

*We will permit applications for certification of devices that do not include the geolocation and database access capabilities, and instead rely on **spectrum sensing** to avoid causing **harmful interference**, subject to a much more rigorous set of tests by our Laboratory in a process that will be open to the public. These tests will include both laboratory and field tests to fully ensure that such devices meet a "Proof of Performance" standard that they will not cause harmful interference.*

*Devices (operating in either mode) will be required to sense TV signals, wireless microphone signals, and signals of other services that operate in the TV bands, including those that operate on intermittent basis, at levels as low as -114 dBm.*

## Sensing difficulty

Detecting digital TV signals is easy due to their embedded pilot tones. Detecting wireless microphones, however, is difficult.

## Wireless microphone usage



*"Going digital would destroy the soul of the music!"*

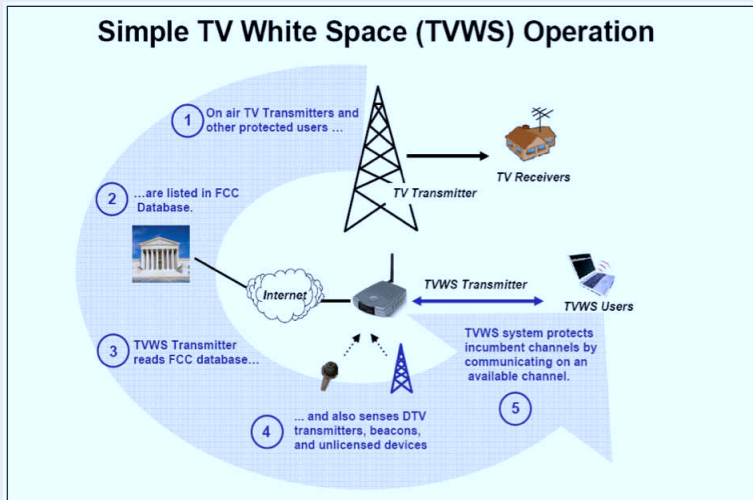
## Sensing wireless microphones (FCC 08-260)

*Wireless microphones will be **protected in a variety of ways**. The locations where wireless microphones are used, such as entertainment venues and for sporting events, can be registered in the database and will be protected as for other services. In addition, channels from 2—20 will be restricted to fixed devices, and we anticipate that many of these channels will remain available for wireless microphones that operate on an itinerant basis. In addition, in 13 major markets where certain channels between 14 and 20 are used for land mobile operations, we will leave 2 channels between 21 and 51 free of new unlicensed devices and therefore available for wireless microphones. Finally, as noted above, we have required that devices also include the **ability to listen to the airwaves** to sense wireless microphones as an additional measure of protection for these devices.*

## Quote (graduate student trying to sense a wireless microphone signal)

"Detecting a wireless microphone is like finding a needle in a haystack. Its signal is very narrow, and it can be anywhere in the spectrum."

# TV White Space DSA



(From "Considerations for Successful Cognitive Radio Systems in US TV White Space", D. Borth et al., Motorola Inc, DySPAN 2008.)



## The database catch 22

Short distance secondary link:

- The database can only be accessed over a wired connection
- If both secondary Tx and Rx need to access the database, they may also communicate over the wired link
- If only one does (can), how does it tell its partner node what frequency to use?

Long-distance secondary link:

- Tx and Rx may have different pictures of the primary user activity. How do they negotiate?
- If the Rx is in a rural area, it may not have database access, at least not very dynamically.

In both cases, CUs may not be aware of other CUs. The **cumulative interference** is not known.

# What is Interference?

## Definition (Interference)

The effect of unwanted energy due to one or a combination of emissions, radiations, or inductions upon reception in an RF communications system, manifested by any performance degradation, misinterpretation, or loss of information which could be extracted in the absence of such unwanted energy.

## Permissible vs. harmful interference

Permissible interference: Defined as **any interference allowed by the FCC**.  
On the other hand, **harmful interference** is prohibited.

## Harmful interference

Topic of heated discussion.

Google July 26, 2010: 263,000 hits for "harmful interference" (in USA).

Google July 30, 2010: 285,000 hits

Two cases with a clear definition:

- UWB: Maximum emission is limited ( $-48.5\text{dBm}/\text{MHz}$ ). More than that is harmful.
- Direct Broadcast Satellite: An increase in unavailability of up to 10% is tolerable (from 0.02% to 0.022%).

But in general?

Definition (HI –

[http://www.its.bldrdoc.gov/fs-1037/dir-017/\\_2541.htm](http://www.its.bldrdoc.gov/fs-1037/dir-017/_2541.htm))

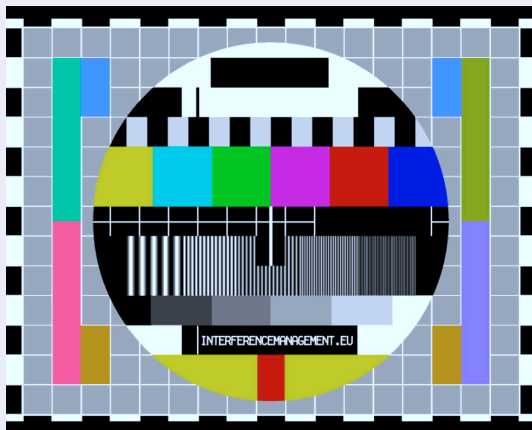
Any emission, radiation, or induction interference that **endangers the functioning** or **seriously degrades, obstructs, or repeatedly interrupts** a communications system, such as a radio navigation service, telecommunications service, radio communications service, search and rescue service, or weather service, operating in accordance with approved standards, regulations, and procedures.

*Note: To be considered harmful interference, the interference must cause **serious detrimental effects**, such as circuit outages and message losses, as opposed to interference that is **merely a nuisance or annoyance that can be overcome by appropriate measures**.*

## HI—European Union (Nov. 29, 2007)

Harmful Interference means interference which degrades or interrupts radiocommunication to an extent beyond that which would reasonably be expected when operating in accordance with the applicable EU or national regulations.

## EU Spectrum Management



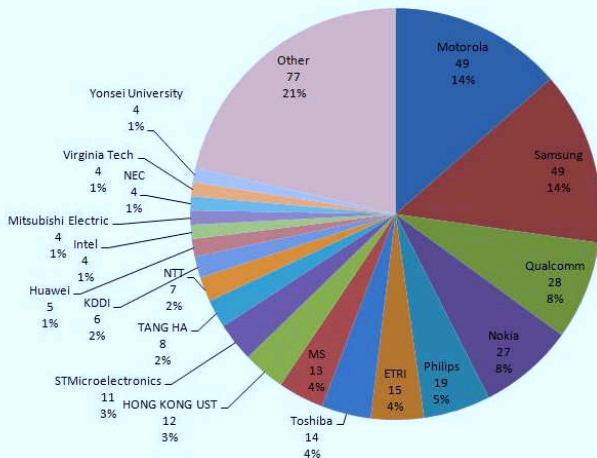
Check [spectrumtalk.blogspot.com/2007/10/european-commission-workshop-on.html](http://spectrumtalk.blogspot.com/2007/10/european-commission-workshop-on.html).

UK: Ofcom at [www.ofcom.org.uk/](http://www.ofcom.org.uk/).

# Patents

## Global Patent Landscape (April 2010; 360 patents issued)

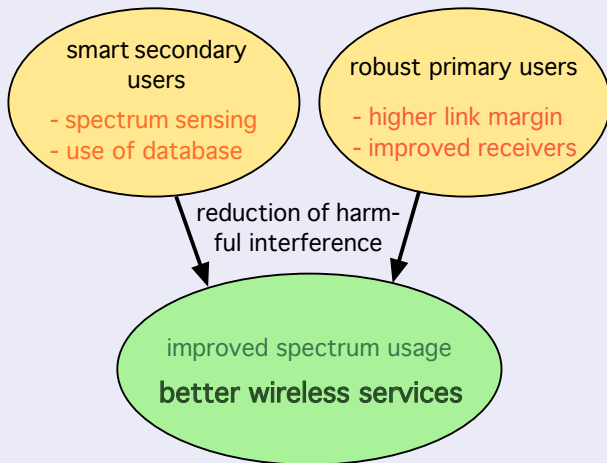
Cognitive Radio Global Patent Landscape by Assignee



# Summary

## Use of White Space

### exploiting white space

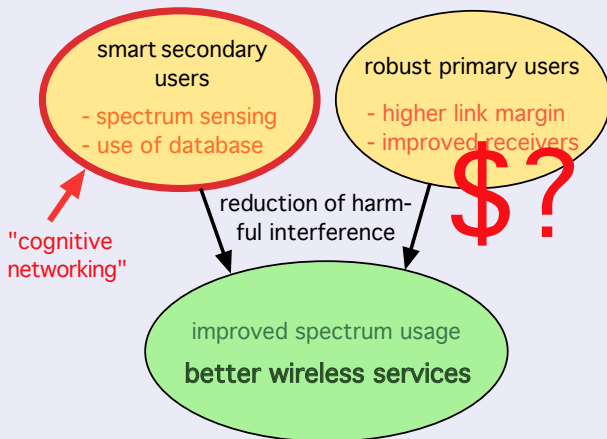




# Summary

## Use of White Space

### exploiting white space

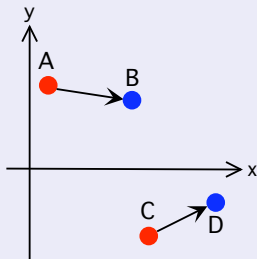


# Interference

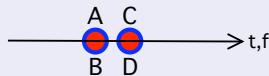
- Interference is the critical issue in wireless networking, in particular in cognitive networking. Physical propagation effects such as shadowing and fading make it hard to characterize and predict.
- Two nodes communicating have a different picture of the situation (hidden or exposed nodes)
- Cognitive networking is essentially a method to better mitigate and manage interference for improved spatial reuse.
- Many physical layer issues (detection, adaptive modulation, frequency switching).
- We focus on interference and its impact on primary users.

# The Network Geometry

Wireless transmissions are separated in space, time, or frequency



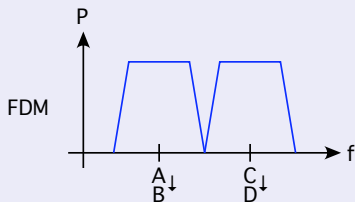
space



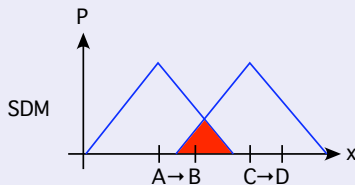
time/frequency

- Separation in time and frequency not sufficient for wireless networks.
- Need for *spatial reuse*. But separation in space is much more challenging.

## Why is spatial reuse hard?



>100dB/decade  
Tx, Rx collocated  
Larger P  $\Rightarrow$  higher R



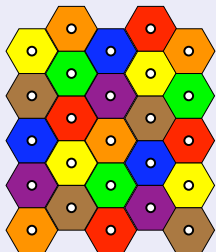
20-40dB/decade (dist.)  
Tx, Rx separated  
SIR independent of P

- There is **interference** between concurrent transmissions.
- Transmitter and receiver have a different picture of the situation.

## Spatial reuse in wireless networks

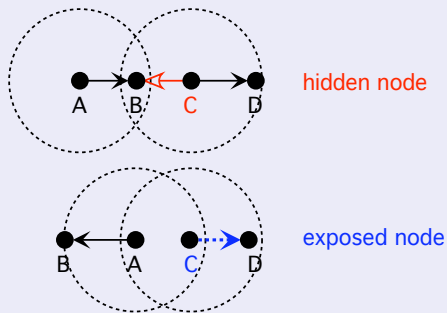
There are several classical channel access schemes. Those requiring coordination among all nodes are not suitable for cognitive networks.

### The cellular solution



Cellular system with frequency reuse factor 1/7

### A sensible solution: CSMA

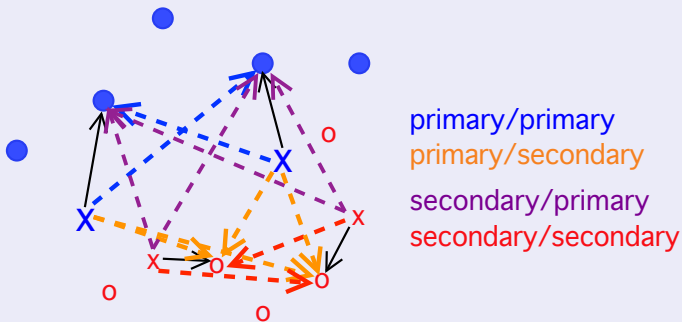


### The simplest solution: ALOHA

Let nodes transmit independently with probability  $p$ .

## Types of interference

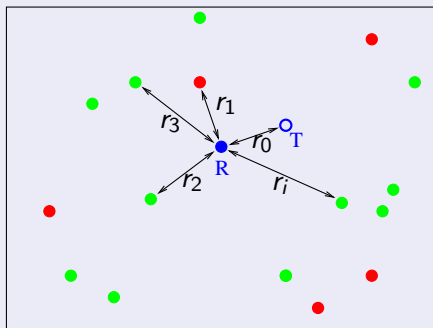
In a cognitive network, there are four types of interference.  
Example with two primary and secondary links each:



We denote the four types as  $I_{pp}$ ,  $I_{ps}$ ,  $I_{sp}$ ,  $I_{ss}$ . The potentially harmful one is  $I_{sp}$ .

How can we characterize these interferences, in the presence of unknown node locations and fading? Stochastic geometry is a promising tool.

## Abstraction: (Part of) a wireless network



- Receiver
- Transmitter
- Inactive node (potential interferer)
- Active node (interferer)

## Basic questions

Given a model for the transmitter (interferer) locations:

- What is the distribution of the interference power at  $R$ ?
- How reliable is the transmission from  $T$  to  $R$ ?
- What is the best rate of transmission?

# Propagation and Physical Layer

## Path loss and fading

If a node transmits at power  $P$  over a distance  $r$ , the received power is

$$S = Phg(r),$$

where:

- $g(r)$  is the large-scale (or mean) path loss law, assumed monotonically decreasing. Typically  $g(r) = r^{-\alpha}$ , where  $\alpha$  is the **path loss exponent**.
- $h$  is the power fading coefficient. We always have  $\mathbb{E}h = 1$ . We usually assume a *block fading model*, where  $h$  changes from one transmission to the next.

Often we consider Rayleigh fading, where  $h$  is exponential:

$$F_h(x) = 1 - \exp(-x), \quad x \geq 0.$$

The amplitude  $\sqrt{h}$  is Rayleigh distributed.



## SINR

With thermal noise of variance  $W$ , the **signal-to-noise ratio (SNR)** is  $S/W = Phg(r)/W$ .

The **interference**  $I$  is the cumulative power from all undesired transmitters.

$$I = \sum_{i \in \mathcal{I}} P_i h_i g(r_i).$$

This leads to the **signal-to-interference-plus-noise ratio (SINR)**

$$\text{SINR} = \frac{Phg(r)}{W + I}.$$

The SINR is our main metric of interest.

## Model for transmission success

$$p_s \triangleq \mathbb{P}(\text{SINR} > \theta).$$

The rate of transmission is smaller than (but can be close to)  $\log_2(1 + \theta)$ .

## Example (Rayleigh block fading with power path loss law)

With  $k$  interferers at known distances  $r_i$  and path loss law  $r^{-\alpha}$ :

$$p_s(r) = \mathbb{P}(S > \theta(W + I)) = \underbrace{\exp\left(-\frac{\theta W}{P} r^\alpha\right)}_{p_s^N} \cdot \underbrace{\prod_{i=1}^k \frac{1}{1 + \theta \frac{P_i}{P} \left(\frac{r}{r_i}\right)^\alpha}}_{p_s^I}$$

## Proof

Let  $S = Phr^{-\alpha}$  be the received power,  $\bar{S} = Pr^{-\alpha}$ , and  $I = \sum_{i=1}^k P_i h_i r_i^{-\alpha}$ .

$$\begin{aligned} p_s &= \mathbb{P}[S > \theta(W + I)] = \mathbb{E}_I \left\{ \exp\left(-\frac{\theta(W + I)}{\bar{S}}\right) \right\} \\ &= \exp\left(-\frac{\theta W}{Pr^{-\alpha}}\right) \cdot \mathbb{E}_I \left\{ \exp\left(-\frac{\theta I}{\bar{S}}\right) \right\} \end{aligned}$$

These are Laplace transforms!  $p_s = \mathcal{L}_W(\theta r^\alpha / P) \cdot \mathcal{L}_I(\theta / \bar{S})$ .

## Remarks

- In a wireless network, there is a lot more uncertainty than fading:  $k$ ,  $r_i$ , perhaps  $P_i$ . There is a need to model uncertainty in the locations of the nodes.
- Let  $I_1$  denote the interference at the receiver. We have

$$\text{SINR}_1 = \frac{Phg(r)}{W + I_1}.$$

Now assume all nodes scale their power by a factor  $a$ . Then  $I_a = aI_1$ , and

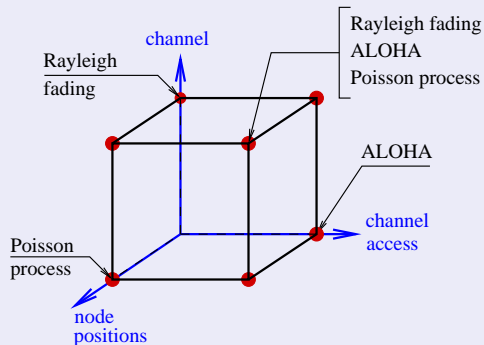
$$\text{SINR}_a = \frac{aPhg(r)}{W + I_a} = \frac{Phg(r)}{W/a + I_1}$$

So, increasing the power **improves the SINR**, since the noise power  $W$  is reduced by  $a$ .

- The noise term  $\exp(-\theta Wr^\alpha/P)$  is less interesting, so we often focus on the SIR only.

# The Uncertainty Cube

## Three dimensions of uncertainty



The **interferer geometry** is determined by the point process (node distribution) and the MAC scheme.

Stochastic geometry permits the characterization of the typical network, using suitable spatial expectations.

# Analysis of Poisson Networks

## Definition (Poisson point process (PPP))

A point process  $\Phi = \{x_1, x_2, \dots\} \subset \mathbb{R}^d$  is Poisson iff

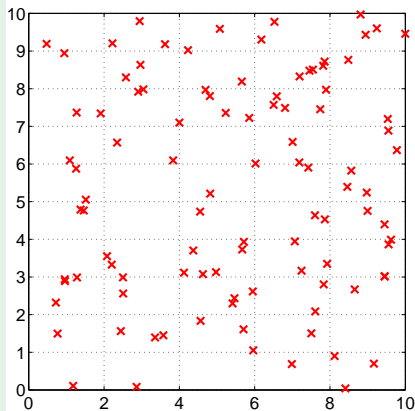
- For all disjoint sets  $B_1, \dots, B_n \subset \mathbb{R}^d$ , the random variables  $\Phi(B_1), \dots, \Phi(B_n)$  are independent.
- For all  $B \subset \mathbb{R}^d$ , the random variables  $\Phi(B)$  are Poisson.

In the stationary case (intensity  $\lambda$ ),

$$\mathbb{P}(\Phi(B) = n) = \frac{(\lambda|B|)^n}{n!} e^{-\lambda|B|}.$$

## Stationary point processes

If  $\Phi$  is stationary,  $\mathbb{E}\Phi(B) = \lambda|B|$  (translation-invariance).

Example (PPP of intensity  $\lambda$ )

Take a Poisson process  $\Phi = \{x_1, x_2, \dots\}$  of constant intensity  $\lambda$  in a square or disk of area  $A$ .

In theory, often  $A \rightarrow \infty$  to avoid boundary issues.

## Two important tools from stochastic geometry

### Probability generating functional (PGFL) for the PPP

For a PPP of intensity  $\lambda$  and a measurable  $0 \leq v \leq 1$ ,

$$G[v] \triangleq \mathbb{E} \prod_{x \in \Phi} v(x) = \exp \left( -\lambda \int_{\mathbb{R}^d} [1 - v(x)] dx \right).$$

### Campbell's theorem for stationary point processes

For measurable  $g(x): \mathbb{R}^d \rightarrow \mathbb{R}^+$ ,

$$\mathbb{E} \left( \sum_{x \in \Phi} g(x) \right) = \lambda \int_{\mathbb{R}^d} g(x) dx.$$

## Laplace transform of the interference

Interference:

$$I \triangleq \sum_{x \in \Phi} h_x \|x\|^{-\alpha},$$

where  $h_x$  is iid with  $\mathbb{E}h = 1$  (fading).

Laplace transform:

$$\begin{aligned} \mathcal{L}_I(s) &= \mathbb{E}(e^{-sI}) = \mathbb{E}_{\Phi, h} \left( e^{-s \sum_{x \in \Phi} h_x \|x\|^{-\alpha}} \right) \\ &= \mathbb{E}_{\Phi} \left( \prod_{x \in \Phi} \underbrace{\mathbb{E}_h(e^{-sh_x \|x\|^{-\alpha}})}_{v(x)} \right). \end{aligned}$$

**Note:** Here we measure the interference at the origin  $o$ , but  $\mathcal{L}_I$  does not depend on the location due to stationarity.



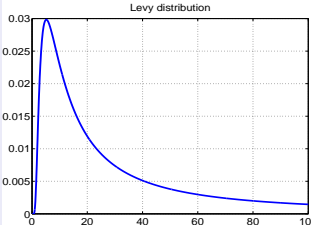
## Laplace transform (cont'd)

If  $\Phi$  is a stationary PPP, using the PGFL,

$$\mathcal{L}_I(s) = G[v] = \exp\left(-\lambda\pi\mathbb{E}(h^\delta)\Gamma(1-\delta)s^\delta\right), \quad 0 < \delta < 1,$$

where  $\delta \triangleq 2/\alpha$ .

## Properties of the interference

- Distribution is *stable* with characteristic exponent  $\delta$ . Pdf only exists for  $\delta = 1/2$ .
  - *I has a heavy tail, no finite moments.*
  - Fading: Only the  $\delta$ -th moment matters.
- 
- As  $\delta \uparrow 1$  (or  $\alpha \downarrow 2$ ), we have  $\mathcal{L}_I(s) \downarrow 0$ , so  $I \uparrow \infty$  a.s.
  - For ALOHA with transmit probability  $p$ , replace  $\lambda$  by  $\lambda p$  (thinning).

## Outage in Rayleigh fading

### Laplace transform for Rayleigh fading

If all interferers are Rayleigh fading,  $\mathbb{E}(h^\delta) = \Gamma(1 + \delta)$ , and

$$\mathcal{L}_I(s) = \exp\left(-\lambda\pi\Gamma(1 + \delta)\Gamma(1 - \delta)s^\delta\right).$$

### Outage for Rayleigh fading desired transmitter

If  $S \sim \exp(1)$ ,

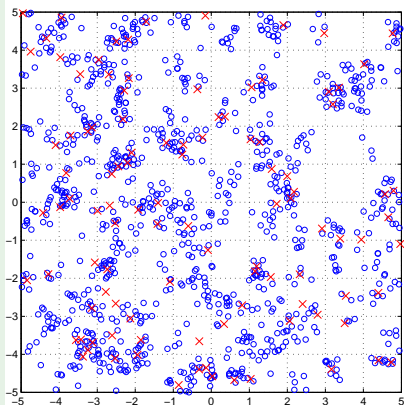
$$p_s = \mathbb{P}(S > I\theta) = \mathbb{E}(e^{-\theta I}) = \exp\left(-\lambda\pi\mathbb{E}(h^\delta)\Gamma(1 - \delta)\theta^\delta\right).$$

Hence  $p_s(\theta) \equiv \mathcal{L}_I(\theta)$ ; the outage  $1 - p_s(\theta)$  is the SIR distribution. So we know more about the SIR than about the interference itself.

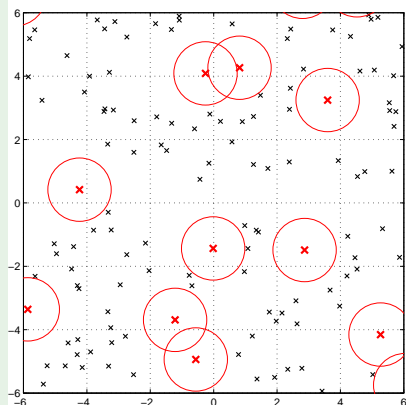
Baccelli et al., "An ALOHA Protocol for Multihop Mobile Wireless Networks", IEEE Trans. Info. Theory, 2006.

# Analysis of General Networks

## Example (Non-Poisson networks)



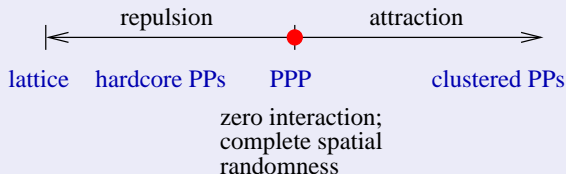
Blue points form a (Poisson)  
cluster process



Red points form a (Matern)  
hard-core process

# Analysis of General Networks

## Point process taxonomy



- Non-Poisson point processes are more difficult to analyze because they lack the independence property. Knowing that there is a point at some locations changes the distribution of the point process.
- **Palm theory** provides the tools to deal with general point processes.
- Hard-core processes are important for CSMA networks **and cognitive networks**.
- Cluster processes are relevant when nodes tend to cluster.

# Weak-interference asymptotics

## Setup

- Take a general motion-invariant PP of intensity  $\lambda$  and a MAC scheme that can tune the intensity of transmitters  $\lambda_t$  from 0 to  $\lambda$ .
- Let  $\eta \triangleq \lambda_t/\lambda$ . What is  $p_s(\eta) = \mathbb{P}(\text{SIR} > \theta)$  for Rayleigh fading?

## Result (Ganti-Andrews-H., 2010)

For all *reasonable* MAC schemes,  $\exists$  unique parameters  $\gamma > 0$  and  $1 \leq \kappa \leq \alpha/2$  s.t.

$$p_s(\eta) \sim 1 - \gamma\eta^\kappa \quad (\eta \rightarrow 0),$$

Moreover,  $p_s(\eta) \geq 1 - \gamma\eta^\kappa$ .

A MAC scheme is reasonable iff  $\lim_{\eta \rightarrow 0} p_s(\eta) = 1$ .

Ganti, Andrews, and H., "High-SIR Transmission Capacity of Wireless Networks with General Fading and Node Distribution", submitted to IEEE Trans. IT.

## Result (from previous slide)

$$p_s(\eta) \sim 1 - \gamma\eta^\kappa \quad (\eta \rightarrow 0)$$

## Discussion

- $\gamma(\alpha, \theta)$  is the *spatial contention* parameter that captures the spatial reuse capability of a network. The smaller the better.
- $\kappa(\alpha)$  is the *interference scaling parameter* and measures the *coordination level* of the MAC. The larger the better.
- For all networks that use ALOHA,  $\kappa = 1$ .
- For lattices with TDMA,  $\kappa = \alpha/2$ .
- CSMA with sensing range  $\Theta(\eta^{-1/2})$  also achieves  $\kappa = \alpha/2$  (hard-core process).
- With fading, the upper bound for  $\kappa$  changes to  $\nu\alpha/2$ , where  $\nu$  depends on the flatness of the fading distribution at zero.

# Summary

## Stochastic geometry ...

- permits the characterization of networks with many sources of uncertainty, most notably in the node location.
- provides concrete results, in particular in the Poisson case, and thus network design insight.

scaling laws

very limited design insight

analysis of networks with fixed geometry

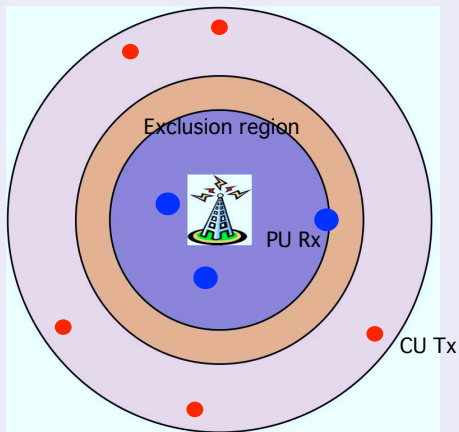
concrete results but no generality

stochastic geometry analysis of the *average* network

generality *and* design insight

# Application to TV White Space

## Setup



Assume CUs are uniformly randomly distributed in the red annulus with density  $\lambda$  (PPP).



## Analysis

Goal: Satisfy the worst-case PU's interference constraint.

Distance between PU and CU at position  $(r, \phi)$ :

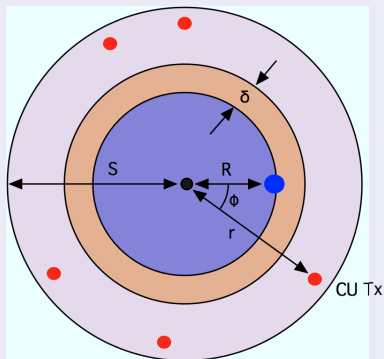
$$d^2(r, \phi) = r^2 + R^2 - 2Rr \cos \phi$$

The CUs are distributed with radial pdf

$$f(x) = \frac{2x}{S^2 - (R + \delta)^2}, \quad R + \delta \leq x \leq S,$$

and the mean number of CUs is

$$n = \lambda \pi (S^2 - (R + \delta)^2).$$



## Analysis

The mean interference is thus, by Campbell's theorem,

$$\mathbb{E}(I) = \lambda P \int_{R+\delta}^S \int_0^{2\pi} \frac{r dr d\phi}{(r^2 + R^2 - 2Rr \cos \phi)^{\alpha/2}},$$

which, for  $\alpha = 4$ , is

$$\mathbb{E}(I) = P \lambda \pi \left[ \frac{(R + \delta)^2}{\delta^2 (2R + \delta)^2} - \frac{S^2}{(S^2 - R^2)^2} \right].$$

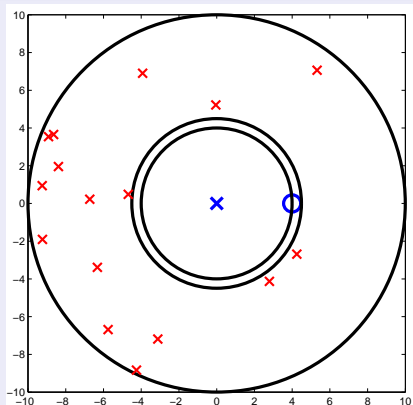
The success probability is

$$p_s = \mathbb{P}(P_{\text{TV}} R^{-\alpha} / I \geq \theta)$$

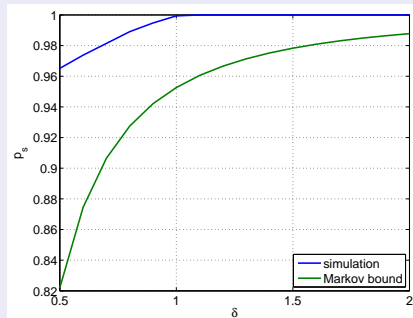
Using Markov's inequality, we obtain

$$p_s \geq 1 - \frac{\mathbb{E}(I) \theta R^\alpha}{P_{\text{TV}}}$$

## Example



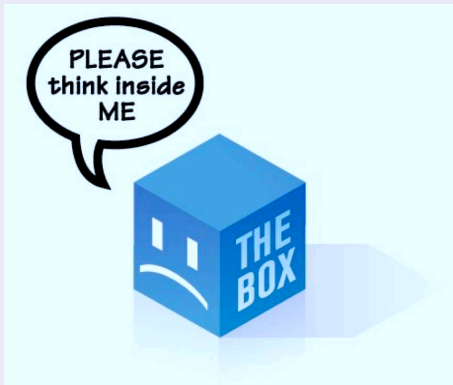
$P_{TV} = 100$ ,  $P = 0.1$ ,  $\lambda = 0.05$ ,  
 $R = 4$ ,  $S = 10$ ,  $\alpha = 4$ ,  $\theta = 4$ ;  
 $n \approx 13$ .



Simulation result and Markov bound as a function of the guard zone width  $\delta$ .

So far so good...

## The white space box



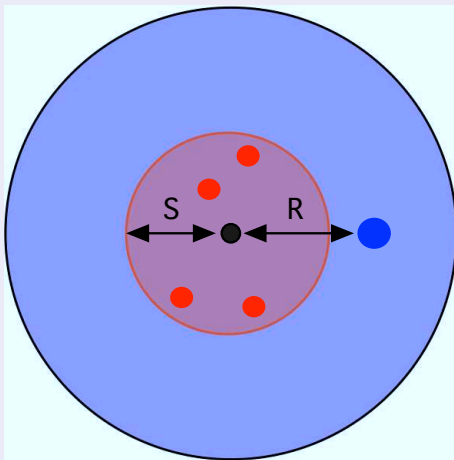
How about...

thinking outside the white space box?



Is the wireless world just black and **white**?

Is there white space **inside** the blue space?



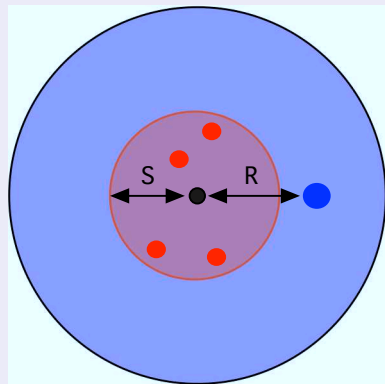
Thinking inside the blue disk...

...but why would we want to put CUs right at the TV station's epicenter??

## Why does it work?

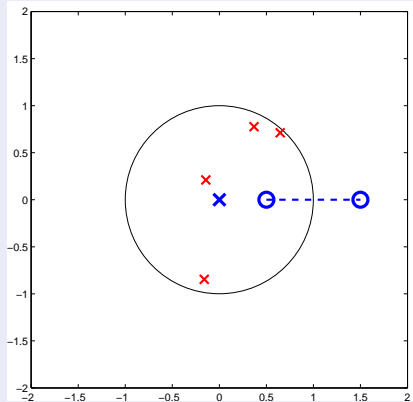
Check the SIR condition!

- Inside the disk of radius  $S$ , the PU's received signal is strong.
- Outside the disk of radius  $S$ , the interference from the CUs is weak.



⇒ Either way, the SIR condition at the PU Rx is met!

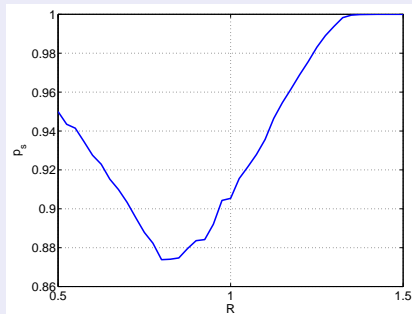
## Example



$$P_{TV} = 100, P = 0.1, \lambda = 1,$$

$$R = [1/2, 3/2], S = 1, \alpha = 4,$$

$$\theta = 4; n \approx 3.$$



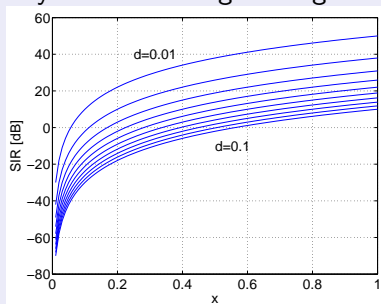
$p_s$  a function of the PU link distance  $R$ .



## How about the secondary receiver?

How is it ensured that the SIR at the secondary receiver is large enough?

- Use small link distances
- Much better: **Use interference canceling techniques!** The TV signal is strong and has a well-defined structure, so it can be subtracted at the secondary receiver, so that there is vanishing interference.



SIR at CU without IC

Interference cancellation is only possible if the interfering signal is stronger. So it is **preferable** to place CUs near the strong TV transmitter!

## Remark on success probabilities

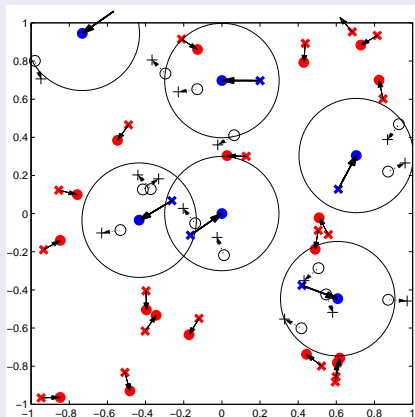
- The success probabilities are **spatial probabilities**. If TV receiver and CUs are static, some TV will never work, others work constantly.
- Only in a mobile scenario, the probabilities can be interpreted temporally also.

# Application to Cognitive Peer-to-Peer Networks

## Bipolar model: Setup

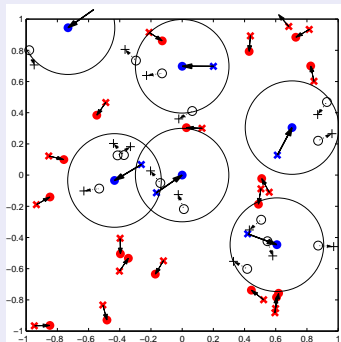
- PU transmitters form a PPP of intensity  $\lambda_p$ .
- CU *potential* transmitters form a PPP of intensity  $\lambda_s$ .
- PU receivers are at distance  $r_p$ .
- CU receivers are at distance  $r_s$ .
- CUs cannot be active if within distance  $D$  of a primary receiver.

The active CUs form a **Poisson hole process**.



## Poisson hole process

- The Poisson hole process with fixed guard zone models a cognitive bipolar peer-to-peer network.
- It is a stationary and isotropic point process.
- Interference compared to the Poisson/Poisson case without guard zone:
  - $I_{pp}$  is unchanged.
  - $I_{ps}$  is smaller, since there is a minimum distance  $D - r_p - r_c$  between a primary Tx and a secondary Rx.
  - $I_{sp}$  is (much) smaller, due to the guard zone  $D$ .
  - $I_{ss}$  changes only due to the smaller intensity of secondary transmitters.  
 $\lambda'_s = \lambda_s \exp(-\lambda_p \pi D^2)$ .



## Interference and outage

The total interference at the typical PU Rx is  $I = I_{\text{pp}} + I_{\text{sp}}$ . Let  $\delta \triangleq 2/\alpha$ .

$$I_{\text{pp}} \triangleq \sum_{x \in \Phi_p} P h_x \|x\|^{-\alpha}$$

$$\mathcal{L}_{I_{\text{pp}}}(s) = \mathbb{E} \exp(-sI) = \exp \left( -\lambda_p \frac{\pi^2 \delta}{\sin(\pi \delta)} P^\delta s^\delta \right).$$

Success probability within PUs:

$$\mathbb{P}(S/I_{\text{pp}} > \theta) = \mathcal{L}_{I_{\text{pp}}}(\theta r_p^\alpha / P) = \exp \left( -\lambda_p r_p^2 \frac{\pi^2 \delta}{\sin(\pi \delta)} \theta^\delta \right)$$

Total success probability: Since  $I_{\text{pp}}$  and  $I_{\text{sp}}$  are negatively correlated:

$$\mathbb{P}(\text{SIR} > \theta) \leq \mathcal{L}_{I_{\text{pp}}}(\theta r_p^\alpha / P) \cdot \mathcal{L}_{I_{\text{sp}}}(\theta r_p^\alpha / P) \quad (\text{by FKG}).$$

But we don't know  $I_{\text{sp}}$ .

## Interference and outage

The critical interference term is  $I_{sp}$ . The point process of transmitting CUs is the Poisson hole process. There are three possibilities to approximate of bound  $I_{sp}$  and the outage probability:

- 1 Approximate the Poisson hole process with a Poisson cluster process by matching first- and second-order statistics. Use known results for Poisson cluster processes to proceed.
- 2 Upper bound the interference by only excluding the CUs outside the reference receiver.
- 3 Approximate the interference by a PPP of secondary transmitters of intensity  $\lambda_s \exp(-\lambda_p \pi D^2)$  outside the guard zone.

We focus on Methods 2 and 3. In both cases, the approximate interference  $\hat{I}_{sp}$  is independent of  $I_{pp}$ , *i.e.*, we're restoring independence.

## Interference and outage

Let  $\hat{I}_{\text{sp}}$  be the interference at the typical PU Rx stemming from a PPP of intensity  $\lambda_s$  outside the guard zone.

$$\mathcal{L}_{\hat{I}_{\text{sp}}}(s) = \exp \left\{ -\lambda_s \pi \left( s^\delta \mathbb{E}_h(h^\delta \gamma (1 - \delta, sh\rho^{-\alpha})) - D^2 \mathbb{E}_h(1 - \exp(-shD^{-\alpha})) \right) \right\}.$$

We know that

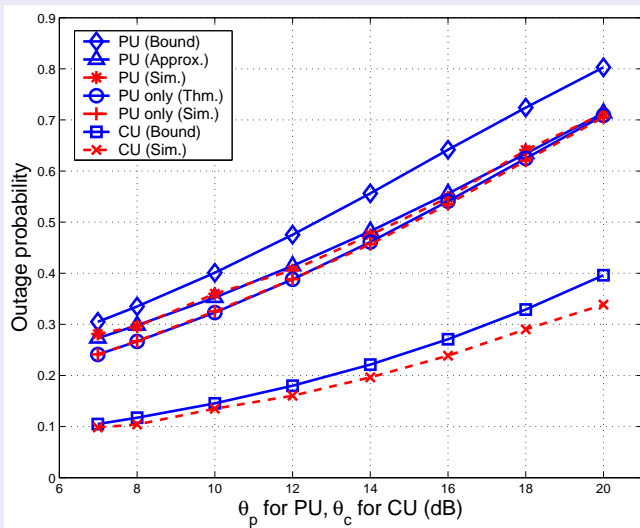
$$\hat{I}_{\text{sp}} \succ I_{\text{sp}}$$

and thus

$$\mathbb{P}(\text{SIR} > \theta) > \mathcal{L}_{I_{\text{pp}}}(\theta r_p^\alpha) \cdot \mathcal{L}_{\hat{I}_{\text{sp}}}(\theta r_p^\alpha)$$

(assuming  $P = 1$ ). Thus the **additional outage** caused by the presence of the CUs is at most  $1 - \mathcal{L}_{\hat{I}_{\text{sp}}}(\theta r_p^\alpha)$ .

## Results

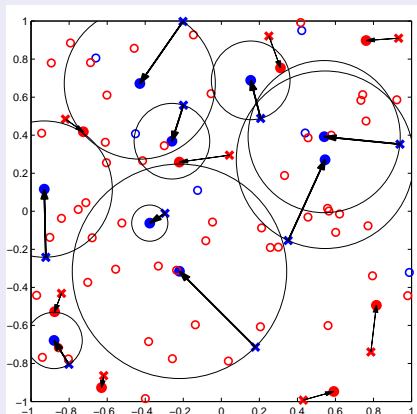




## Nearest-neighbor model: Setup

- PUs form a PPP of intensity  $\lambda_p$ .
- CUs form a PPP of intensity  $\lambda_s$ .
- PUs apply ALOHA with prob.  $p_p$ . Tx finds nearest node as its receiver.
- CUs cannot be active if within distance  $D_i$  of a primary receiver.
- Other CUs use ALOHA with prob.  $p_c$  and transmit to nearest neighbor.

The guard zone  $D_i$  is a random variable with known distribution.



## Interference and outage

From the probability generating functional for PPPs it follows that:

The intensity of secondary transmitters is  $\exp(-\rho_p)$ .

This is independent of  $\lambda_p$ , since a larger  $\lambda_p$  implies smaller guard zones. In fact,  $\mathbb{E}(D^2) = \lambda_p^{-1}$ .

Similar approximations as in the bipolar case lead to good bounds.

## Exclusion regions around transmitters

- Exclusion regions around receivers can make sense if their locations are known (database).
- With a sensing-based approach, only transmitters can be detected.
- With guard zones around the primary transmitters, the primary receivers suffer from increased interference  $I_{sp}$ , as the effective guard zone radius reduces to  $D - r_p$ .  $I_{pp}$  and  $I_{ss}$  remain the same, and  $I_{ps}$  decreases.
- If a receiver acknowledges packet reception, its presence can also be detected. A CU can match transmitter-receiver pairs and transmit concurrently with a PU transmitter if the PU receiver is on the other side.

## The mutual nearest-neighbor model

- In the previous nearest-neighbor model, the receiver may not be able to acknowledge, since there may be another node nearby.
- To prevent ACK collision, the **mutual-nearest-neighbor** transmission protocol may be applied. Here, nodes form nearest-neighbor pairs if they are mutual nearest neighbors. The fraction of nodes thus paired is 62%.
- The resulting point process of transmitters thus has maximum density 31%, and it is more regular than a PPP.

# Outlook

## Ongoing and future work

- Software-defined radio
- (Collaborative) detection and learning
- Standardization (IEEE 802.22)
- Economic aspects (spectrum leasing, pricing) and game theory
- Legal aspects: how to detect and punish cheaters? The “hit and run” radio problem.
- Database issues
- Ruling on TV white space
- Network protocols, in particular for CUs (including Tx-Rx coordination)

## Concluding remarks

- Cognitive radio enables the transition from "spectrostatics" to "spectrodynamics".
- Space is the critical resource; the network geometry greatly affects the interference and thus the performance of cognitive networks.
- Need to consider all potential CUs, not just one.
- Stochastic geometry permits the analysis of interference and outages in many scenarios where nodes are randomly distributed.
- The problem of white spaces is not a black and white problem. Wireless transmissions offer many gray areas, especially if advanced receiver technologies are available.
- "FCC rules are like Maxwell's equations"

Cognitive networks pose multi-faceted challenges: Technical, economic, legal, and policy issues.

## Cognitive Radio Policy and Regulations

- U.S. National Broadband Plan ([www.broadband.gov](http://www.broadband.gov))
- Ofcom Statement on Cognitive Devices ([stakeholders.ofcom.org.uk/binaries/consultations/cognitive/statement/statement.pdf](http://stakeholders.ofcom.org.uk/binaries/consultations/cognitive/statement/statement.pdf))
- IEEE 802.22 WG on Enabling Rural Broadband Wireless Access Using Cognitive Radio Technology ([www.ieee802.org/22/](http://www.ieee802.org/22/))
- Proceedings of the Dynamic Spectrum Access (DySPAN) conferences

## Stochastic Geometry

- Haenggi, Andrews, Baccelli, Dousse, and Franceschetti, "Stochastic Geometry and Random Graphs for the Analysis and Design of Wireless Networks", IEEE J. on Sel. Areas in Comm., Sept. 2009.
- Haenggi and Ganti, "Interference in Large Wireless Networks", Foundations and Trends in Networking, NOW Publishers, 2008.
- Baccelli and Blaszczyszyn, "Stochastic Geometry and Wireless Networks", Foundations and Trends in Networking, NOW Publishers, 2009.